Wyższa Szkoła Bankowa
we Wrocławiu

WSB UNIVERSITY
RESEARCH
JOURNAL

DEMATEL-based Ranking Approaches

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Abstract

DEMATEL is an efficient tool for the identification of cause-effect relations which link the objects considered. A dynamical development of its popularity in recent years has resulted in numerous efforts to eliminate its weaknesses and to extend its application potential. For example, one of the attempts involved transforming it into a universal decision-making support tool aimed at weighting and ranking objects seamlessly. A critical review of the existing weighting and ranking extensions in DEMATEL is, therefore, presented in the paper. Conclusions about the usefulness of the available extensions have been drawn based on the results of an exemplary analysis.

Introduction

DEMATEL represents a tool for supporting decision-making developed in the '70s of the last century in order to implement the research project dedicated to the identification of cause-effect relations among the problems facing today's world (Fontela and Gabus, 1976). Over time it appeared that the method could be used successfully to solve problems coming from other fields. The application of the method was accompanied by the need to adapt the computational procedures, to eliminate its drawbacks and to extend its application potential. Numerous advantages of this tool have had the effect that people have been increasingly interested in its application as a universal tool supporting decision-making, e.g. used for eliciting weights depicting relative importance (weighting) and ranking of objects. Currently, the method employs a few differentiated techniques of weighting and ranking, which, however, can ultimately lead to different results. This justifies the focus of this paper on the issue of usefulness of particular techniques designed for reliable weighting and ranking of objects.

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Techniques of Weighing and Ranking under the DEMATEL Method

Introduction

In the practice of decision-making support, a variety of weighting and ranking methods find their application. They can be divided into three groups ((Ishizaka and Nemery 2013). The first one comprises the methods stemming from the so called American school of decision-making support which apply Multi-Attribute Value Theory (MAVT) and Multi-Attribute Utility Theory (MAUT). The second group includes methods employing the concept of outranking, with the third group being made up of the methods which use the concept of the reference level. Amongst the tools mentioned above, Analysis of Net Processes (ANP) (Saaty, 1996) is to be distinguished, for it provides one of few, and, in addition, comprehensively verified procedures of weighting and determining on this basis the ranking of objects while taking into account interrelations among them. It is this possibility of taking into account the complex links existing among objects that makes the ANP a valuable tool for solving the issues of contemporary decision-making. In order to obtain correct results of the analysis, it is necessary to use the relation structure among the objects. Yu-Ping Ou Yang et al. (2008) showed that the DEMATEL method could successfully be used in determining this structure. Further to that, they also proposed to adapt the ANP computational procedure to a direct processing of the information on total influence of objects, obtained using the DEMATEL method to determine the weights of objects interacting with one another. Their approach was named DANP, DEMATEL-based ANP. It consists in forming matrix S, similar to the ANP supermatrix, on the basis of appropriately modified total-influence matrix T

representing the outcome of the DEMATEL method application (Fontela and Gabus, 1976):

$$\mathop{\forall}_{i=1,2\dots n} \quad \mathop{\forall}_{j=1,2\dots n} \quad s_{ij} = \frac{t_{ji}}{\sum_{k=1}^{n} t_{jk}}, \quad (1)$$

where: n is the number of interrelated objects, t_{ji} denotes the element of the matrix T depicting the total influence of the j-th object on the i-th object, with s_{ij} representing an appropriate component of the matrix S.

Next, we determine the limiting matrix Slim on the basis of the matrix S, using the procedure characteristic for ANP

$$\mathbf{S}_{\lim} = \lim_{k \to \infty} \mathbf{S}^k \cdot \quad (2)$$

Matrix Slim consists of n identical columns. The elements of the individual columns depict normalized weights of the objects which form an n-element-vector z. Those weights provide the basis for ranking the objects.

DANP has been recognised as an exemplary technique of weighting and ranking objects. Nevertheless, in practice, other techniques are also applied. They can be divided into two groups. The techniques included in the first group use complete information on total influence depicted by matrix T. Thus, DANP belongs to this group too. The second group encompasses methods representing the attempts aimed at simplifying the process of weighting and ranking objects through the application of partial information of total influence. The methods included in both of these groups have been outlined below.

The Application of Complete Information on Total Influence

The earliest attempt to use the information on total influence of objects for their weighting and ranking was made by Hiroyuki Tamura and Katsuhiro Akazawa (2005). It stems from

the theory of cognitive maps (Axelrod 1976). It involves the application of the following formula:

$$z = (I + T)y, \qquad (3)$$

in which: z denotes the vector of the object weights, I is a unit matrix, while y is the vector of the object weights showing initial opinions as to the importance of objects.

Vector y consists of weights indicating the importance of objects which is referred to the importance of the object initially recognized as the most important. The components of the vector are determined subjectively.

This is not conducive to the reliability of the above method.

The application of the procedure (2) requires labour-consuming raising the matrix S to a power. Therefore, in the book (Ginda, 2015) a technique designed to facilitate bringing weights z closer is proposed:

$$\underset{i=1,2\dots n}{\forall} \quad z_i = \frac{\sum_{k=1}^{n} s_{ij}}{n} . \qquad (4)$$

The last technique of eliciting object weights was developed by Hiete et al. (2012). It involves the application of analytic hierarchy process – AHP (Saaty, 1980) for the initial bringing closer of the weights of objects p. These weights are then corrected using the results produced by the application of the DEMATEL method:

$$\underset{i=1,2\dots n}{\forall} \quad z_i = p_i \ w_i , \qquad (5)$$

where the correcting weights are described by the following formula:

$$\underset{i=1,2\dots n}{\forall} \quad w_i = 1 - \frac{1}{n \max_i \left\{ \max_j \left\{ t_{ij} \right\} \right\}} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} . \qquad (6)$$

Let us notice that the influence on the final values of the object weights z, apart from the results obtained based on the DEMATEL method, has the subjective estimation of the weights,

resulting from the application of the AHP method.

Using Partial Information on Total Influence

As the result of utilizing the DEMATEL method, for every object, we obtain the values of position indicators (s+) and relation indicators (s−). The first one shows the importance of the object arising from the interrelation with other objects, whereas the other indicator shows the influence of the object on other objects. These indicators represent partial information about total influence of objects which has been employed in a few proposals regarding weighting and ranking of objects. Some utilize both indicators simultaneously, while others use only one indicator. Let us note here that the originators themselves of the DEMATEL method suggested the possibility of a two-factor ranking of objects using both indicators (Fontela and Gabus, 1976). Their proposal, however, referred to two independent contexts of ranking objects. The first one shows relations among objects, and the second one – the influence of objects. This possibility has also been used by other researchers, for example, Mirosław Dytczak (2008). However, in this paper, we are interested in utilizing the DEMATEL method solely for general weighting and ranking of objects. Therefore, further in the paper, we will limit ourselves to this kind of application of indicators s+ and s−.

Mirosław Dytczak and Grzegorz Ginda (2008) proposed to use the indicator s− , seen as a stimulant, for ranking objects. The above indicator takes on non-negative and negative values. This makes its application for weighting objects impossible. No such drawback is present in indicator s+, for it takes on only non-negative values. That is why Supratik Dey et al. (2012) proposed the following technique

where the indicator was employed for weighting and ranking objects:

$$\underset{i=1,2\dots n}{\forall} \quad z_i = \frac{s_i^+}{\sum_{i=1}^{n} s_i^+} . \tag{7}$$

A similar proposal was also formulated by Selcuk Cebi (2013):

$$\underset{i=1,2\dots n}{\forall} \quad z_i = \frac{s_i^+}{\sum_{i=1}^{n} s_i^{+2}} . \tag{8}$$

The first attempt to use both indicators simultaneously with a view of weighting and ranking was illustrated by Doraid Dalalah (2009). With this in mind, he depicted weights z using Euclidean point distance representing objects on the plane of both of the indicators at the origin (s+, s−):

$$\underset{i=1,2\dots n}{\forall} \quad z_i = \sqrt{\left(s_i^+\right)^2 + \left(s_i^-\right)^2} . \tag{9}$$

Moreover, Andrzej Kobryń (2014) has recently proposed the following formula:

$$\underset{i=1,2\dots n}{\forall} \quad z_i = \frac{1}{2}\left(s_i^+ + s_i^-\right) . \tag{10}$$

## Comparing the Results of the Application of Different Techniques of Weighting and Ranking

Comparison Methodology

With a view to compare the results of the application of different techniques of weighting and ranking objects, the work of Ranjan et al. (2015) was utilized. The work uses the DEMATEL method to identify the role and interdependencies occurring among six efficiency factors of engineering departments of a university. The first one, marked with the symbol FS, represents the force of the department arising from the staff numbers and the number of students enrolling at the department. The factor marked as RP represents the impact of the number of publications by the department's employees. The two other factors, SS and DT, represent, respectively: the number of graduates and completed doctoral studies. The last two factors, TN and OC, refer to costs. The first of them shows a unit annual cost of retaining staff. Moreover, the other one corresponds to a unit cost of retaining a student.

Table 1. Evaluation of the direct-influence intensity of the factors.

Factor	FS	RP	SS	DT	TN	OC
FS	0	3	4	3	4	1
RP	2	0	1	3	1	1
SS	2	1	0	1	1	3
DT	2	3	1	0	1	2
TN	3	2	3	2	0	1
OC	1	1	3	3	1	0

[Source: Author's own study based on (Ranjan et al. 2015)]
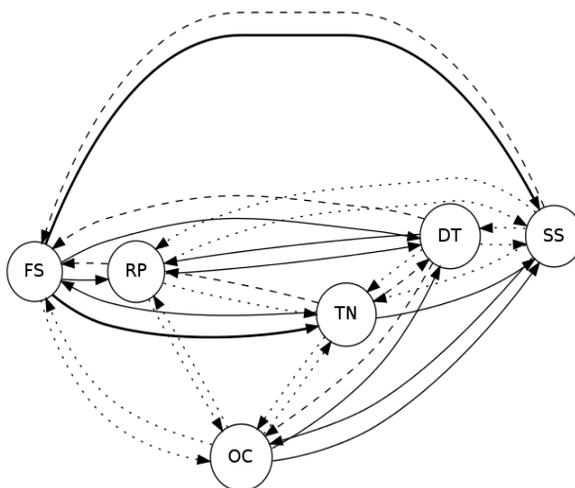
Fig. 1. The structure of the factor direct influence

[Source: Author's own study based on (Ranjan et al. 2015)]

In order to evaluate the intensity of direct influence of the factors, a five-degree scale of direct influence 0-4 was used. Thanks to its use, experts evaluated the direct-influence relations. The evaluations of the relations were gathered in Table 1, and Figure 1 shows a directed graph depicting a direct-influence structure corresponding to them. The differentiation of the intensity of the direct influence was depicted using a variety of curves. The highest level of intensity is shown by a thick continuous line – curve. The continuous line of normal thickness denotes 3 on the scale, dash-line – level 2, and dot-line – level 1. As the result of the application of the DEMATEL method, we obtain the total-influence structure indicated by matrix T. Table 2 demonstrates the elements of this matrix and the values, resulting from them, of position and relation indicators corresponding to the individual factors.

Table 2. The depiction of the total-influence structure.

T	FS	RP	SS	DT	TN	OC	i	S_i^+ [-]	S_i^- [-]
FS	0,340	0,505	0,595	0,543	0,490	0,347	1	4,793	+0,847
RP	0,310	0,205	0,279	0,396	0,223	0,224	2	3,633	−0,359
SS	0,303	0,252	0,228	0,290	0,221	0,336	3	3,896	−0,637
DT	0,325	0,386	0,301	0,252	0,235	0,290	4	4,081	−0,503
TN	0,431	0,381	0,468	0,413	0,218	0,284	5	3,803	+0,587
OC	0,264	0,267	0,395	0,398	0,220	0,182	6	3,391	+0,063

[Source: Author's own study based on (Ranjan et al. 2015)]

Table 3. Weights and ranking of the factors obtained based on correlation (3)

Factor	FS	RP	SS	DT	TN	OC
I	1	2	3	4	5	6
zi [-]	0,171	0,168	0,185	0,193	0,135	0,147
Rank	3	4	2	1	6	5

[Source: Author's own study]

For the techniques of weighting and ranking of factors which describe correlations (3,5), it was planned to conduct a sensitivity analysis of the weighting and ranking results to the initially adopted values of the weights. For this purpose, three different sets of weights y and p were utilized. They differ in terms of the factor order.

The first of the weight sets applied for correlation (3) is identical to the set z obtained through the application of DANP. As the result of its application, we obtain the weights and ranking of the factors presented in Table 3. Factor DT is ranked at the very top, followed in succession by: SS, FS, RP, OC and TN.

On the basis of the weights showed in Table 3 we obtain vector $\mathbf{y}^{(+)}$ of the following form:

$$\mathbf{y}^{(+)} = [0{,}886 \ \ 0{,}871 \ 0{,}961 \ \ 1 \ \ 0{,}701 \ \ 0{,}760]^{\mathrm{T}}. \ (11)$$

The second set $\mathbf{y}^{(+/-)}$ indicates the same initial importance of the factors, which means that it contains only ones. The third set $\mathbf{y}^{(-)}$ comprises the same weight values as set (11). However, they were assigned to the factors in a reverse order.

In the case of the application of formula (5) for determining the weights

Table 4. Data and results of the AHP application for the determination of the weight set p(+)

A	FS	RP	SS	DT	TN	OC	I	$p_i^{(+)}$ [-]
FS	1	2	1/2	1/2	3	4	1	0,182
RP	1/2	1	1/2	1/2	3	4	2	0,145
SS	2	2	1	1/2	4	5	3	0,250
DT	2	2	2	1	4	5	4	0,315
TN	1/3	1/3	1/4	1/4	1	1/2	5	0,053
OC	1/4	1/4	1/4	1/5	2	1	6	0,056

[Source: Author's own study]

of the first set p$^{(+)}$, AHP was utilized. The values of the evaluations presented in Table 4 were taken into account. They result in a similar ranking of the factors as the one produced when using DANP.

The sets $\mathbf{p}^{(+/-)}$ and $\mathbf{p}^{(-)}$ were obtained based on $\mathbf{p}^{(+)}$ in a similar way as the sets $\mathbf{y}^{(+/-)}$ and $\mathbf{y}^{(-)}$ based on $\mathbf{y}^{(+)}$. The sets of the weight initial values are illustrated in Fig. 2.

The Results of Factor Weighting and Ranking

Fig. 3 presents the results of the application of the particular techniques. In addition, the ranking of factors was illustrated resulting from using the indicator s– (Dytczak and Ginda 2008). Table 5 shows the normalized weight values and the factor rankings resulting from them. The positions of the factors in the rankings are given by the numbers in round brackets.
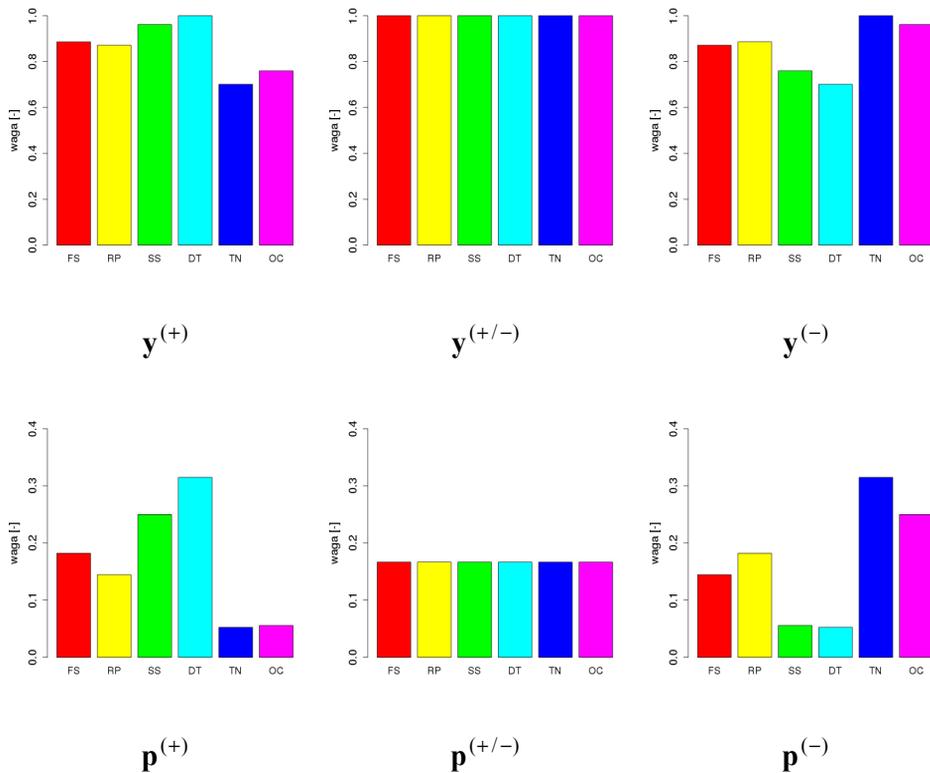

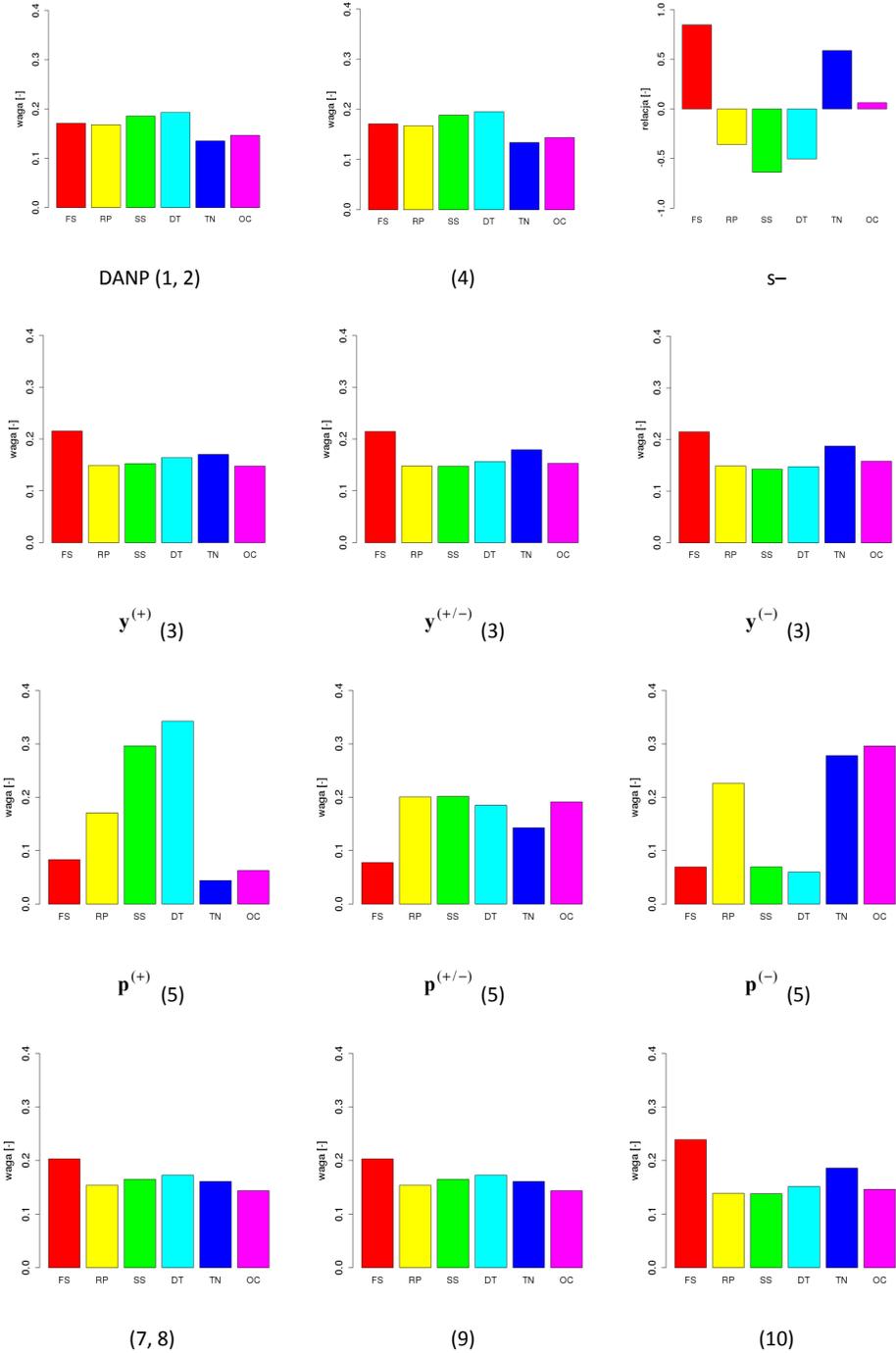**Fig. 2 The sets of the initially adopted values of weights**

[Source: Author's own study]

Fig. 3 Comparison of the results of the various techniques of weighting and ranking of factors

[Source: Author's own study]

Table 5. The comparison of the results of weighting (and ranking) of factors

Formula	FS	RP	SS	DT	TN	OC
(3)	0,171 (3)	0,168 (4)	0,186 (2)	0,193 (1)	0,135 (6)	0,147 (5)
(4)	0,171 (3)	0,167 (4)	0,189 (2)	0,195 (1)	0,134 (6)	0,144 (7)
y(+) (1)	0,216 (1)	0,149 (5)	0,153 (4)	0,164 (3)	0,170 (2)	0,148 (6)
y(+/−) (1)	0,215 (1)	0,148 (5)	0,148 (5)	0,157 (3)	0,180 (2)	0,153 (4)
y(−) (1)	0,215 (1)	0,149 (4)	0,143 (6)	0,148 (5)	0,187 (2)	0,158 (3)
p(+) (5)	0,083 (4)	0,171 (3)	0,296 (2)	0,343 (1)	0,044 (6)	0,063 (5)
p(+/−) (5)	0,078 (6)	0,201 (2)	0,202 (1)	0,185 (4)	0,143 (5)	0,192 (3)
p(−) (5)	0,070 (4)	0,226 (3)	0,070 (4)	0,060 (6)	0,278 (2)	0,296 (1)
(7, 8)	0,203 (1)	0,154 (5)	0,165 (3)	0,173 (2)	0,161 (4)	0,144 (6)
(9)	0,204 (1)	0,153 (5)	0,166 (3)	0,173 (2)	0,162 (4)	0,142 (6)
(10)	0,239 (1)	0,139 (5)	0,138 (6)	0,152 (3)	0,186 (2)	0,146 (4)
s−	+0,847 (1)	−0,359 (4)	−0,637 (6)	−0,503 (5)	+0,588 (2)	+0,063 (3)

[Source: Author's own study]

The results of an exemplary analysis suggest that the weighting and ranking techniques employed in the DEMATEL method lead to differentiated results. None of the techniques alternative to DANP has produced similar weight values. The differentiation of the results, however, does not only refer to the weight values, but also to the forms of the factor rankings obtained on their basis.

Amongst the techniques using complete information on total influence of factors, only when applying formula (5) was it possible to obtain a ranking of the factors similar to that obtained through the DANP application. Yet, in order to achieve this, a correct estimation of the initial weight values proved to be necessary. Moreover, technique (3) failed completely, as, no matter what the initially adopted set of weight values was, it produced rankings which clearly diverged from the appropriate ordering of the factors.

None of the techniques utilising partial information on factor total-influence did allow obtaining an appropriate order of the factors. Techniques (7,8), which apply only the position indicator, and technique (9) produce similar sets of weight values and factor rankings. Despite the application of both indicators, technique (10) produces weight values and factor ranking different from the results produced by technique (9). Additionally, this ranking is very similar to the ranking obtained as the result of using solely the relation indicator s−. Thus, the results presented show that DANP is by far the best technique for a reliable determination of weights and rankings of objects under the DEMATEL method. Its application makes the use of formula (4) easier, since the results of the exemplary analysis show that when utilizing it we can produce similar effects.

Summary and Conclusions

The results of the exemplary analysis allow for the following conclusions. Firstly, the application of the techniques of object weighting and ranking, proposed by different authors, aimed at expanding the potential of the DEMATEL method produces varied results.

Secondly, the simplification of calculations thanks to the application of partial information on total influence of objects not only leads to errors in weight estimation, but also to incorrect rankings of objects. Therefore, in order to determine correct weight values and ranking of objects, one should utilize full information on total influence of objects.

Thirdly, aiming at limiting the influence of the decision-maker's subjectivity when weighting and ranking objects, one should utilize the possibilities provided by the integration of the computation procedures of the DEMATEL and ANP methods. Furthermore, the qualitative nature of the DEMATEL method (Dytczak and Ginda, 2013) is conducive to simplifying the process of determining object rankings thanks to the application of the similar technique of transforming the full information on total influence of objects (4)

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Procedury rangowania w metodzie DEMATEL

Abstrakt

Metoda DEMATEL stanowi narzędzie identyfikacji związków przyczynowo--skutkowych. Zainteresowanie metodą w ostatnich latach znajduje odzwierciedlenie w licznych publikacjach z różnych dziedzin. Duże zainteresowanie DEMATELem pozwoliło na liczne modyfikacje tej metody, usunięcie mankamentów i poszerzenie potencjału aplikacyjnego. W rezultacie takich działań powstało uniwersalne narzędzie wspomagania decyzji umożliwiające ważenie i rangowanie kilkoma różnymi sposobami. W artykule dokonano krytycznego przeglądu takich sposobów. Porównano ich rezultaty i sformułowano wnioski dotyczące ich przydatności.

Słowa kluczowe: DEMATEL, rozwój, zastosowanie, ważenie, rangowanie