



## Forecasting economic dynamics of Germany using conditional models (1992-2014)

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### Abstract

A great diversity characterizes economic dynamics of Germany over a long period of time. This refers to many time series: in some periods, they show large volatility which then moves into stability and stagnation phase, generating specific difficulties in a long-term forecasting of economic dynamics. The aim of the research is the attempt to determine the prognostic efficiency of conditional modelling and to answer the question whether or not conditional errors are significantly smaller than the unconditional ones in long-term forecasting. The research showed that conditional errors (root mean square errors RMSE) of an ex- post forecast did not differ significantly from the unconditional RMSE. The decreasing RMSE of the ex-post forecast for Germany's individual economic processes (with the assumption that an intercept occurs in the ARMA procedure) was correlated more strongly with the procedure of filtering economic time series than with the application of the conditional maximum likelihood method (ML) and robust procedures. The relationship between a decreasing RMSE of the ex-post forecast and the application of conditional ML methods occurs in ARMAX forecasts (with exogenous processes) for data filtered with Hodrick - Prescott (HP) filter. It is worth pointing out that a relatively high prognostic efficiency of the robust (resistant) estimation of quantile regression occurs for the economic series linearized with the help of the TRAMO/SEATS method.

**Keywords:** robust procedures, quantile regression, ARMA, ARMAX, Hodrick - Prescott filter, TRAMO/SEATS.  
**JEL:** C32, E32

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### Introduction

Defining the conditional expectation of data-generating random processes whose conditional variance does

not vary over time boils down to the issue of the mean squared error<sup>1</sup>. The

<sup>1</sup> See: (Athreya, Lahiri, 2006), (Kosiorowski, 2012)

regression analysis for random variables (X, Y) consists in predicting Y based on the observations of X. Therefore, one should find function  $f$ , such that  $Y = f(X)$ . The mean squared error is usually assumed to be the criterion which is the measure of the accuracy of estimation Y. Assuming that the expected value of the prediction Y is finite, one proves that there exists function  $f_0$  which minimises the mean squared prediction error<sup>2</sup>.

It seems *prima facie* reasonable to assume that the volatility of economic dynamics of time series over long periods of time increases the unconditional mean squared errors of their prediction. However, economic time series are not generated solely by random processes. Were it the case, any attempts at making predictions would prove futile, for random processes cannot be predicted. That does not mean that an appropriately employed "reception strategy" may consider a meaningless, "random" communication to be "relevant" and "structured". In this case, however, the organization of the communication, which has been imposed by the reception strategy, emerges only "on the side" of the receiver who somewhat succumbs to the "hallucination" ..... of "information"<sup>3</sup> Yet, economic processes are not random isolates as they are inter-connected through a variety of relations (causal, probabilistic, fuzzy, reflexive, etc.). Moreover, it appears that the efficiency of predictions is to some extent conditional on applying the procedures of pre-whitening data.

The size of the mean squared prediction errors may also be affected by outliers occurring in the time series considered. This kind of data may occur in individual time series. One can also view an entire time series as an outlier in a given set (family) of the time

series<sup>4</sup>. Their impact on the results of a regression analysis is to a large extent conditional on the computation procedures employed. In order to remove the impact of the outliers on the computation results, they are frequently removed from the time series. Furthermore, the procedures correcting outliers are used (averaging, filtration, etc.). In the processes which generate a high percentage of outliers in all observations in the period considered, such interference is very likely to result in seeming regressions. In literature, attention is therefore paid to the application of resistant statistical procedures in economic studies minimizing mean squared errors<sup>5</sup>.

The aim of the study on economic dynamics of Germany presented below is the verification of the hypotheses asserting that variance and deviating observations significantly change the mean squared errors of forecasting the dynamics of economic time series.

## Research methods

In a preliminary pre-whitening of monthly data (270 data covering the period between April 1992 and September 2014, 23 time series of Germany, calculated as a month corresponding to the month of the previous year) - in order to remove a seasonal component from the original data, the X-12-ARIMA procedure was used in gretl software 1.10.2 (MS Windows x86\_64). In the result, a component adjusted seasonally (nazwa [name]\_s) was obtained. The seasonally adjusted time series were then subject to augmented Dickey-Fuller unit-root test in

<sup>4</sup> Lopez - Pintado, Romo 2006; Lopez - Pintado, Romo 2009

<sup>5</sup> See: Brandt, 1999; Chow, 1995; Chukwu, 2003; DeLurgio, 1998; Edgeworth, 1888; Ekonometria, 2003; Greene 2000; Jajuga, 1993; Kosiorowski, 2012; Madala, 2006); Statystyczna analiza danych, 2009; Studenmund, 2001.

<sup>2</sup> Althreya, Lahiri, 2006

<sup>3</sup> Lem, 2009, p. 269

gretl software. The test showed that there were no grounds to reject the null hypothesis: “a unit-root is present” for all the time series adjusted seasonally. In order to remove the stochastic trend, the Hodrick-Prescott filter was used in gretl software (the lambda coefficient for monthly data = 14400). The cyclical component obtained was recorded as *hp\_nazwa\_s*. The seasonally adjusted data were subject to the TRAMO/SEATS analysis using gretl software. As a result, adjusted data *nazwa\_s\_xl* and deviating values IO (innovational outliers), additive outliers AO, LS (level shift) and TC (temporary changes) were obtained<sup>6</sup>. For the thus obtained time series, the ARMA forecasting model was used (assuming that only intercept and covariance matrix parameter via Hessian will be included). The accuracy of the quantitative forecasts was assessed using the ex post mean squared error in the verification interval. The interval of the empirical verification of the forecasting was determined for the period February 2014 –October 2014 (eight months)<sup>7</sup>. As a non-seasonal order AR and non-seasonal order MA in the ARMA model (for data *nazwa\_s* and *hp\_nazwa\_s*), the quantities set out in *armax* package 0.92 (author: Yi - Nung Yang) of gretl software were adopted. For data *nazwa\_s\_xl*, the non-seasonal and seasonal order AR and MA were employed, determined automatically in the TRAMO/SEATS procedure of gretl software. Next, for individual data sets (cleared of seasonal variations, with Hodrick-Prescott stochastic trend being removed, and cleared of linearised series), the forecasts for the industry production dynamics of Germany were determined with the help of the ARMAX models (applying exogenous process), using

<sup>6</sup> See: Marona, Martin, Yohai 2006.

<sup>7</sup> See: Prognozowanie gospodarcze, 2001; Clements, Hendry, 2001; Clements, Hendry, 2004; Chatfield 2000.

the appropriate and conditional maximum likelihood method<sup>8</sup> and quantile regression models. Similar to forecasting time series, an ex post mean squared error was computed in the verification interval February-October 2014. Deviations were computed for all the time series<sup>9</sup>.

Subject to the analysis were the following time series of the German economy (the period = 1992-2014, the number of data = 270; the data have been presented as chain indices: month related to the month of the previous year):

The data were obtained from the monthly magazine Statistisches Bundesamt „Wirtschaft und Statistik” and the publications of the tests of the economic situation in industry and construction by Ifo-Institut für Wirtschaftsforschung in Munich.

## Empirical Studies

The standard deviations of the time series with Hodrick-Prescott stochastic trend being removed are smaller than the standard deviations of the data after having used the X-12 ARIMA procedure. Moreover, the standard deviations of the time series obtained in the TRAMO/SEATS analysis are smaller than the data with seasonality removed in 16 cases, and for the data which were filtered with Hodrick-Prescott filter only in 8 cases (the data used in the graph below have been included in Table 1 of the Statistical Attachment).

The level of the standard deviations of the time series does not seem to be related to the occurrence of outliers

<sup>8</sup> The appropriate method of maximum likelihood uses Kalman filter algorithm, whereas the conditional method of maximum likelihood uses BHHH algorithm ((E. R. Berndt, B. H. Hall, R. E. Hall, J.A. Hausman; Kufel, 2007; Maddala, 2006; Berndt, Hall, Hall, Hausman, 1974).

<sup>9</sup> Kufel, 2007.

**Table 1. Selected monthly time series of German economy (April 1992 – September 2014)**

No	Time series	Symbol
1	DAX stock index	dax
2	Hard coal mining	coal
3	Steel production	steel
4	Cement production	cement
5	Electricity production	electri
6	Production of passenger cars	cars
7	Total industrial production	prod
8	Production of capital goods	inv
9	Production of raw materials	row
10	Production of consumer goods	consum
11	Industry turnover (in nominal terms)	turn_n
12	Industry turnover (in real terms)	turn_r
13	Price dynamics of industrial goods	cen_dyn
14	Job vacancy	vacan
15	Industrial employment	employ
16	Total unemployment	unemploy
17	Export	export
18	Import	import
19	Economic situation forecast for industry	pr_prz
20	Diagnosis of economic situation in industry	d_prz
21	Industrial goods price forecast	pr_cen
22	Economic situation forecast for construction	pr_bud
23	Diagnosis of economic situation in construction	d_bud

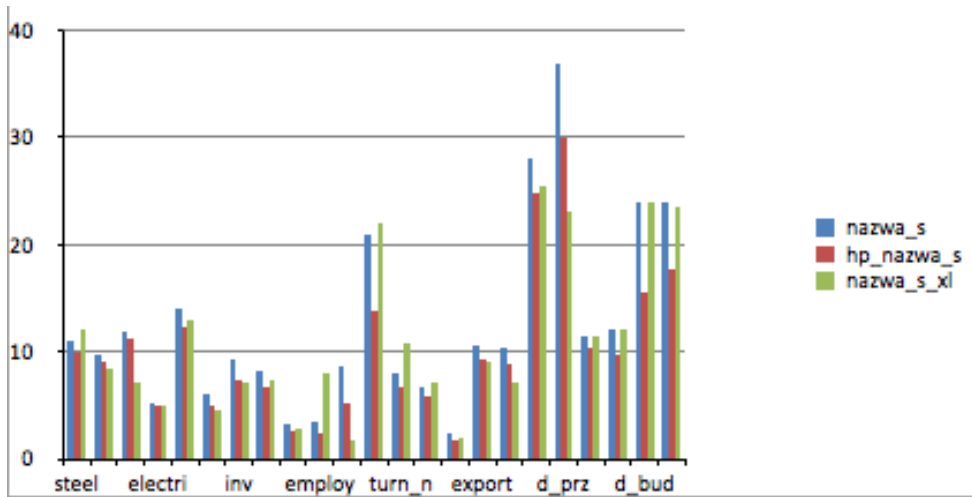
Source: Author's own study

in these time series. For example, the high variance (standard deviation = 28,043) of the dynamics of forecast indicators for economic activity of German industry is not related to the occurrence of a great number of outliers in a given time series (IO in January 2009 and in October 2008)<sup>10</sup>:

Moreover, for the industrial goods price dynamics showing the smallest standard deviation in the period considered, the TRAMO/SEATS analysis determined seven outliers (IO in November 2008, in January and May 2014; AO in July 2009; LS in January 2009; TC in April 2003 and April 2014)<sup>11</sup>

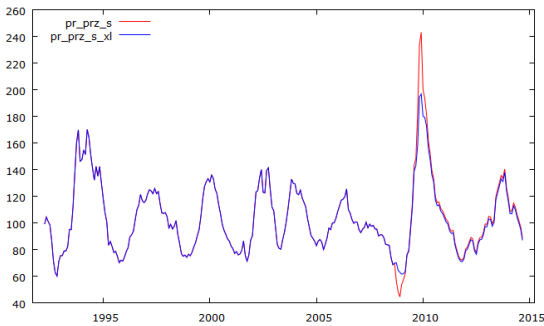
<sup>10</sup> See Table 2 of the Statistical Attachment

<sup>11</sup> See Table 2 of the Statistical Attachment



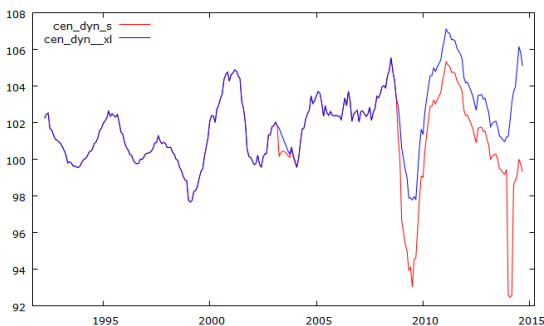
**Fig. 1.** Time series standard deviations after removing seasonality (name\_s), after applying Hodrick-Prescott filter (hp\_nazwa\_s) and after being subject to the TRAMO/SEATS analysis (04/1992 - 09/2014).

Source: Wirtschaft und Statistik, Ifo-Institut, www.borse.de (accessed on 30 June 2015)



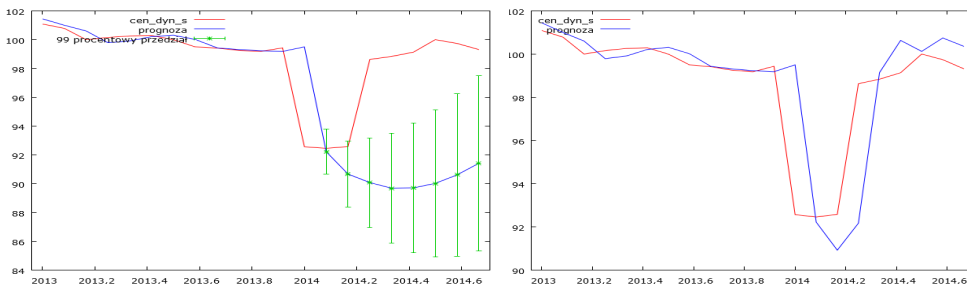
**Fig. 2.** The dynamics of the German economic situation forecast for industrial production (monthly data 1992/04 – 2014/09) smoothed in the X-12 ARIMA and TRAMO/SEATS analyses.

Source: Author's own study



**Fig. 3.** Germany's industrial goods price dynamics (monthly data 1992/04 – 2014/09) smoothed in the X-12 ARIMA and TRAMO/SEATS analyses.

Source: Author's own study



**Fig. 4 - 5. The dynamic (the left graph) and static forecast (the right graph) in the verification interval 02 – 09/2014 of the industrial goods price dynamics after removing the seasonal component in the X-12 ARIMA analysis (using the appropriate maximum likelihood method)**

Source: Author’s own study

The ex post root mean squared error of the forecast (RMSE) in the verification interval (02-09/2014) of the time series of economic dynamics of Germany after removing from them the seasonal component (X-12 ARIMA)<sup>12</sup> did not show large differences with respect to the forecasts obtained using the appropriate (ordinary) and conditional method of the maximum likelihood of the ARMA model. Only for eight time series (per 23 in total) did the RMSE of the “conditional” forecasts prove to be smaller than the RMSE of the “ordinary” forecasts (coal, cement, electri, prod, row, consum, unemploy, vacan). For the industrial goods price dynamics and industrial goods price forecast, the RMSE was computed additionally for a static (and not dynamic) forecast outside the sample range (cen\_dyn\_s\_stat, pr\_cen\_s\_stat). The dynamic forecasts for those time series appeared to have little likelihood in the verification interval:

The mean squared error of the forecast for the time series of Germany’s economic dynamics, after having removed the seasonal component and stochastic trend (using Hodrick-Prescott filter),

showed relatively bigger differences with respect to the “conditional” and “ordinary” forecasts for the time series of the steel production dynamics, the industry economic situation forecast and the DAX stock exchange:

The RMSE of the conditional forecasts proved to be smaller than the RMSE of the ordinary forecasts for ten time series of the economic dynamics of Germany (steel, electri, cars, prod, row, pr\_prz, d\_prz, pr\_bud, d\_bud, dax).

The ex post mean squared errors of the forecast of the economic dynamics time series of Germany (after being transformed using the TRAMO/SEATS method) obtained using the ARMA model<sup>13</sup> are smaller (than the “ordinary” forecasts errors) for 14 time series when applying the conditional maximum likelihood method: coal, cement, electri, prod, inv, row, unemploy, vacan, turn\_r, cen\_dyn, export, import, pr\_prz, pr\_cen. While removing the seasonal component and stochastic trend from the time series considered, we can observe an increase in the number of the time series whose “conditional” forecasts show smaller mean squared errors than

<sup>12</sup> See Table 3 of the Statistical Attachment

<sup>13</sup> See Table 5 of the Statistical Attachment



**Fig. 6 - 11. The ARMA dynamic forecast in the verification interval 02 – 09/2014 of the steel production dynamics, the economic situation forecast for German industry and the DAX index, after removing stochastic trend using the appropriate (the left graph) and conditional (the right graph) maximum likelihood method.**

Source: Author's own study

**Table 2 Ex post root mean squared errors of forecast using the appropriate (ordinary) and conditional maximum likelihood method (February 2014 – September 2014)**

Time series	ex post root mean squared error of forecast	
	Appropriate ML method	Conditional ML method
nazwa_s	4,908724	5,127520
hp_nazwa_s	4,544892	4,609084
nazwa_s_xl	4,094528	4,034166

Source: Author’s own study

the “ordinary” forecasts. Although the average level of those errors is reduced, the mean squared errors using the conditional maximum likelihood method were smaller on average than the errors of the forecast using the appropriate ML method only for the data subject to transformation by the TRAMO/SEATS method. The forecasts of the time series dynamics of the raw material production in Germany are a good illustration of how an ex post forecast error gets reduced when moving from the appropriate to the conditional ML method,

together with the series transformations by the X-12 ARIMA method, Hodrick-Prescott filter and the TRAMO/SEATS method. The forecasts of the time series dynamics of industrial production and electricity production (showing a relatively smaller error for the conditional ML method) have minimal errors for the data with the cyclical component being removed and without the trend (hp\_nazwa\_s). The estimation of the ARMAX prediction models and quantile regression is only partially consistent with the results achieved so far.

**Table 3. Ex post mean squared error of forecast using the appropriate (ordinary) and conditional maximum likelihood method (February 2014 – September 2014)**

ARMAX models	RMSE (appropriate)	RMSE (conditional)	Time series
Model 1	1,7838	1,6121	nazwa_s
Model 2	1,7183	1,5154	hp_nazwa_s
Model 3	1,8353	1,7797	nazwa_s_xl
Quantile regression	RMSE		
Model 4	2,0079		nazwa_s
Model 5	1.8246		hp_nazwa_s
Model 6	1,7927		nazwa_s_xl

Source: Author’s own study





**Fig. 12 - 17.** Ex post ARMA forecasts of the raw material production dynamics in Germany (02-09/2014) after having removed seasonal fluctuations, stochastic trend and after linearization using the TRAMO/SEATS method and the application of the appropriate (the left graph) and conditional (the right graph) maximum likelihood method.

Source: Author's own study

In the ARMAX models and quantile regression, the endogenous variable was the industrial production dynamics of Germany, whereas 22 variables were exogenous variables (processes linked to industrial production, employment, sale, international exchange, economic situation forecasting and stock exchange cycles, see Table 1). An ex post minimum forecasting error occurred

in Model 2 (data with seasonality and stochastic trend removed):

Function evaluations: 104  
 Evaluation of gradient: 38

Model 2: ARMAX estimation, using observations 1992-06-2014:01 (N=260)  
 Estimation using BHHH method (conditional ML)  
 Dependent variable (Y): hp\_prod\_s

**Table 4. ARMAX estimation of independent variables hp\_variable\_s.**

	<b>coefficient</b>	<b>standard error</b>	<b>z</b>	<b>p value</b>
<b>const</b>	0,007311	0,140263	0,05212	0,9584
<b>phi_1</b>	0,048957	0,05295	0,9246	0,3552
<b>phi_2</b>	0,058898	0,063205	0,9318	0,3514
<b>theta_1</b>	0,07785	0,102854	0,7569	0,4491
<b>theta_2</b>	0,086456	0,10729	0,8058	0,4204
<b>hp_steel_s</b>	0,038826	0,020052	1,936	0,0528 *
<b>hp_coal_s</b>	-0,0161641	0,014449	-1,119	0,2633
<b>hp_cement_s</b>	0,048616	0,011135	4,366	1,26E-05 ***
<b>hp_electri_s</b>	0,051306	0,021425	2,395	0,0166 **
<b>hp_cars_s</b>	0,036776	0,011687	3,147	0,0017 ***
<b>hp_inv_s</b>	0,278991	0,024701	11,29	1,39E-29 ***
<b>hp_row_s</b>	0,046506	0,027397	1,698	0,0896 *
<b>hp_consum_s</b>	0,118566	0,044674	2,654	0,008 ***
<b>hp_employ_s</b>	-0,0600114	0,071695	-0,8370	0,4026
<b>hp_unempl_s</b>	-0,00183356	0,034713	-0,05282	0,9579
<b>hp_vacan_s</b>	0,016234	0,014933	1,087	0,277
<b>hp_turn_n_s</b>	-0,574603	0,457011	-1,257	0,2086
<b>hp_turn_r_s</b>	0,686252	0,46274	1,483	0,1381
<b>hp_cen_dyn_s</b>	0,717697	0,506308	1,418	0,1563
<b>hp_export_s</b>	-0,0324103	0,020998	-1,543	0,1227
<b>hp_import_s</b>	0,003727	0,021088	0,1767	0,8597
<b>hp_pr_prz_s</b>	0,002578	0,012675	0,2034	0,8388
<b>hp_d_prz_s</b>	-0,00279557	0,01001	-0,2793	0,78
<b>hp_pr_cen_s</b>	0,062655	0,025979	2,412	0,0159 **
<b>hp_pr_bud_s</b>	0,035397	0,01715	2,064	0,039 **
<b>hp_d_bud_s</b>	-0,00786290	0,012528	-0,6276	0,5303
<b>hp_dax_s</b>	0,006406	0,011791	0,5433	0,5869

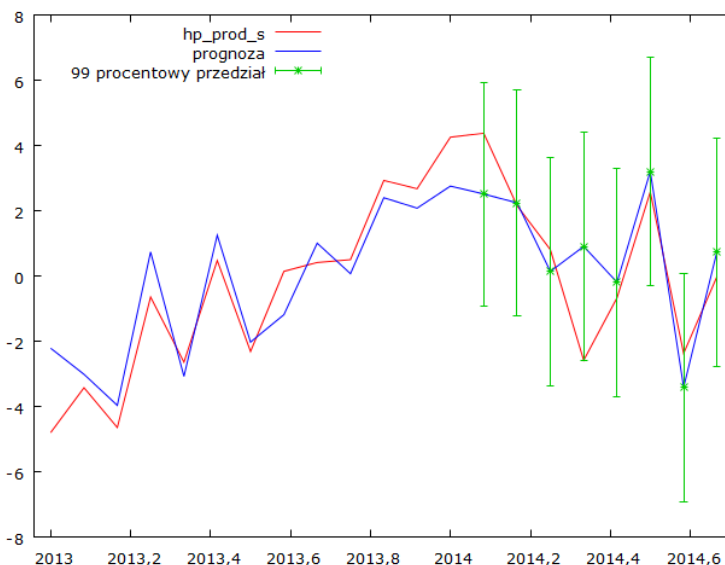
Source: Author's own study

Dependent variable's arithmetic mean	0,045066
Dependent variable's standard deviation	5,064565
The mean of random errors	0,000585
Standard deviation of random errors	1,333011
Likelihood logarithm	443,6584
Akaike information criterion	943,3169
Schwarz bayesian criterion	1043,016
Hannan-Quinn criterion	983,3972

**Table 5 ARMAX estimation of dependent variable hp\_prod\_s (Model 2)**

part	real	imaginary	module	frequency
-----				
AR				
Root 1	3,7258	0,0000	3,7258	0,0000
Root 2	-4,5570	0,0000	4,5570	0,5000
MA				
Root 1	-0,4502	-3,3710	3,4010	-0,2711
Root 2	-0,4502	3,3710	3,4010	0,2711
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Source: Author's own study



**Fig. 18. Ex post ARMAX forecast of the industrial production dynamics of Germany (02-09/2014), after having removed seasonal fluctuations and stochastic trend using the conditional ML method.**

Źródło: opracowanie własne

The models obtained were tested for normality of residual distribution and residual graphs Q– Q (quantile- quantile) were generated. In none of the models were there any grounds for rejecting the null hypothesis: “the empirical distribution function has a normal distribution”;

Frequency distribution for uhat2, observations 3-262

Interval number = 17, the mean = 0,000585178, standard deviation = 1,40813

Null hypothesis: the empirical distribution function has a normal

distribution. Doornik-Hansen test (1994) – transformed skewness and kurtosis: Chi-square (2) = 196,805 with p value 0,00000

In the quantile regression models (Model 4 – Model 6) the minimum ex post error occurred in Model 6 (data linearised using the TRAMO/SEATS procedure):

Model 6: Estimation quantile Estimator, using observations 1992:04-2014:01 (N = 262)

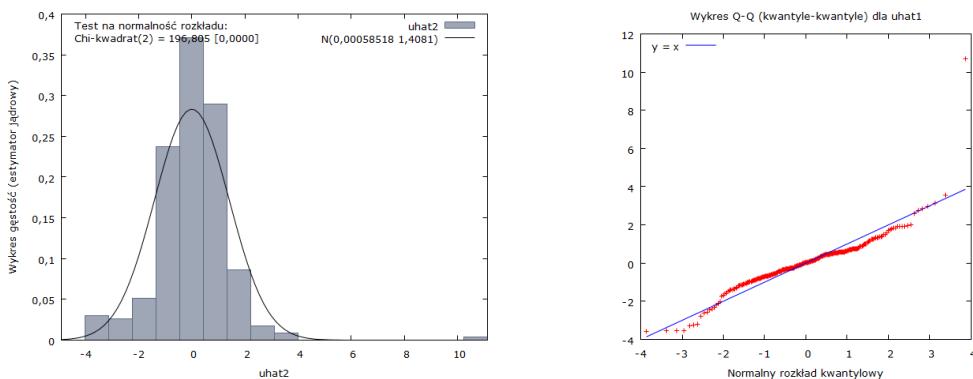
Dependent variable(Y): prod\_s\_xl, tau = 0,5

Asymptotic standard errors (IID errors)

**Table 6 Frequency distribution for Model 2**

intervals	mean	number	frequency	cumulative
< -3,1314	-3,577	7	2,69%	2,69%
-3,13140 ÷ -2,24030	-2,6859	6	2,31%	5,00%
-2,24030 ÷ -1,34910	-1,7947	12	4,62%	9,62% *
-1,34910 ÷ -0,45792	-0,90351	55	21,15%	30,77% *****
-0,45792 ÷ 0,43326	-0,01233	86	33,08%	63,85% *****
0,43326 ÷ 1,32440	0,87884	67	25,77%	89,62% *****
1,32440 ÷ 2,21560	1,77	20	7,69%	97,31% **
2,21560 ÷ 3,10680	2,6612	4	1,54%	98,85%
3,10680 ÷ 3,99800	3,5524	2	0,77%	99,62%
3,99800 ÷ 4,88910	4,4435	0	0,00%	99,62%
4,88910 ÷ 5,78030	5,3347	0	0,00%	99,62%
5,78030 ÷ 6,67150	6,2259	0	0,00%	99,62%
6,67150 ÷ 7,56270	7,1171	0	0,00%	99,62%
7,56270 ÷ 8,45380	8,0082	0	0,00%	99,62%
8,45380 ÷ 9,34500	8,8994	0	0,00%	99,62%
9,34500 ÷ 10,2360	9,7906	0	0,00%	99,62%
>=10,2360	10,682	1	0,38%	100,00%

Source: Author's own study



**Fig. 19-20. Distribution normality test and Q-Q chart for Model 2 using the conditional ML method.**

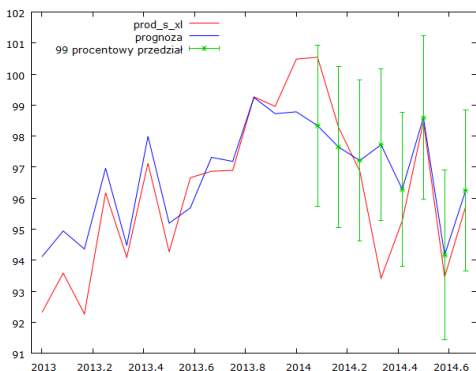
Source: Author’s own study

**Table 7. The results of the quantile regression of the dependent variable prod\_s\_xl (Model 6)**

nazwa_s_xl	coefficient	standard error	Student’s t	p value
steel_s_xl	0,079088	0,011419	6,926	3,94e-011 ***
coal_s_xl	-0,0101808	0,00957	-1,064	0,2885
cement_s_xl	0,084524	0,014294	5,913	1,15e-08 ***
electri_xl	0,052285	0,020676	2,529	0,0121 **
cars_s_xl	0,013057	0,009256	1,411	0,1597
inv_s_xl	0,253775	0,02177	11,66	3,53e-025 ***
row_s_xl	0,069733	0,020355	3,426	0,0007 ***
consum_s_xl	0,124415	0,039321	3,164	0,0018 ***
unempl_s_xl	-0,00814542	0,013022	-0,6255	0,5322
employ_s_xl	0,224689	0,056453	3,98	9,13e-05 ***
vacan_s_xl	0,021406	0,005542	3,863	0,0001 ***
turn_n_s_xl	0,000391	0,01355	0,02885	0,9770
turn_r_s_xl	0,057217	0,020122	2,843	0,0048 ***
cen_dyn_xl	0,036413	0,066361	0,5487	0,5837
export_s_xl	0,009736	0,015596	0,6243	0,5330
import_s_xl	-0,0699114	0,015211	-4,596	6,96e-06 ***
pr_prz_s_xl	0,010845	0,005201	2,085	0,0381 **
d_prz_s_xl	0,019162	0,006001	3,193	0,0016 ***
pr_cen_s_xl	0,030599	0,013152	2,327	0,0208 **
pr_bud_s_xl	0,022208	0,010807	2,055	0,0410 **
d_bud_s_xl	-0,00535156	0,005482	-0,9762	0,3300
dax_s_xl	-0,00115374	0,004629	-0,2492	0,8034

Source: Author’s own study

Median of dependent variable 101,3516  
 Standard deviation of dependent variable 4,594462  
 Absolute sum of squares 238,2022  
 Residual sum of squares 474,3636  
 Likelihood logarithm - 418,6557  
 Akaike information criterion 881,3114  
 Schwarz bayesian criterion 959,8150  
 Hannan-Quinn criterion 912,8637  
 Frequency distribution for uhat3, observations 1-262  
 Interval number = 17, the mean = -0,115469, standard deviation = 1,4007



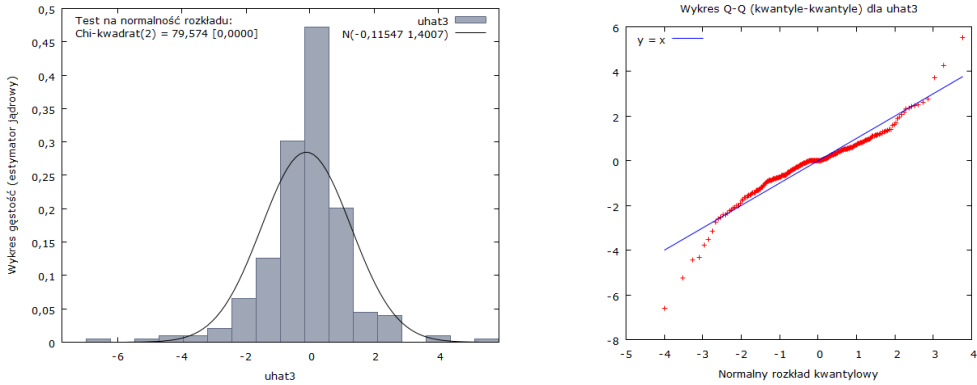
**Fig. 21. Ex post forecast of industrial production dynamics of Germany (02-09/2014) obtained using quantile regression for data linearised by the TRAMO/SEATS method.**

Source: Author's own study

**Table 8 Frequency distribution for Model 6**

intervals	mean	number	frequency	cumulative
< -6,2404	-6,62	1	0,38%	0,38%
-6,2403 - -5,4810	-5,8607	0	0,00%	0,38%
-5,4810 - -4,7217	-5,1014	1	0,8%	0,76%
-4,7217 - -3,96124	-4,3421	2	0,76%	1,53%
-3,9624 - -3,2031	-3,5827	2	0,76%	2,29%
-3,2031 - -2,4438	-2,8234	4	1,53%	3,82%
-2,4438 - -1,6844	-2,0641	13	4,96%	8,78% *
-1,6844 - -0,92511	-1,3048	25	9,54%	18,32% ***
-0,92511 - -0,16579	-0,54545	60	22,90%	41,22% *****
-0,16579 - 0,59353	0,21387	94	35,88%	77,10% *****
0,59353 - 1,3529	0,97319	40	15,27%	92,37% *****
1,3529 - 2,1122	1,7325	9	3,44%	95,80% *
2,1122 - 2,8715	2,4918	8	3,05%	98,85% *
2,8715 - 3,6308	3,2512	0	0,00%	98,85%
3,6308 - 4,3901	4,0105	2	0,76%	99,62%
4,3901 - 5,1495	4,7698	0	0,00%	99,62%
>= 5,1495	5,5291	1	0,38%	100,00%

Source: Author's own study



**Fig. 22-23. Distribution normality test and chart Q - Q for Model 6.**

Source: Author's own study

Null hypothesis: the empirical distribution function has a normal distribution. Doornik-Hansen test (1994) – transformed skewness and kurtosis: Chi-square (2) = 79,574 with p value 0,00000

**Conclusions**

The comparative analysis of the ex post mean squared forecast errors has showed that conditional forecast errors do not differ significantly from the unconditional ones. The ex-post decreasing forecast error for Germany's individual economic processes (assuming that intercept occurs in the ARMA procedure) is correlated more strongly with

the procedure of filtering economic time series than with the application of the conditional maximum likelihood method and robust procedures. The relationship between the decreasing ex post mean squared error and the application of the conditional ML method occurs in the ARMAX procedure (with exogenous processes) with the minimum conditional forecast error occurring for data filtered using Hodrick - Prescott (HP) filter. It is worth pointing out that a relatively high prognostic efficiency of the robust estimation procedure of quantile regression occurs for the data linearized with the help of the TRAMO/SEATS method.

## Statistical Attachment

**Table 1. Standard deviations of time series after removing seasonality (nazwa\_s), after using Hodrick-Prescott filter (hp\_nazwa\_s) and after being subject to the TRAMO/SEATS analysis (04/1992 - 09/2014)**

nazwa_s	standard deviation	hp_nazwa_s	standard deviation	nazwa_s_xl	standard deviation
steel	11,097	hp_steel_s	10,025	steel_s_xl	12,189
coal	9,7615	hp_coal_s	9,1053	coal_s_xl	8,4709
cement	11,865	hp_cement_s	11,193	cement_s_xl	7,0549
electri	5,2875	hp_electri_s	4,8831	electri_xl	4,9216
cars	14,114	hp_cars_s	12,22	cars_s_xl	12,993
prod	6,0316	hp_prod_s	4,9964	prod_s_xl	4,6041
inv	9,196	hp_inv_s	7,4278	inv_s_xl	7,0891
row	8,1494	hp_row_s	6,7741	row_s_xl	7,2948
consum	3,1664	hp_consum_s	2,6303	consum_s_xl	2,7462
employ	3,5138	hp_employ_s	2,2892	unempl_s_xl	7,9275
unempl	8,6298	hp_unempl_s	5,2648	employ_s_xl	1,6796
vacan	20,968	hp_vacan_s	13,797	vacan_s_xl	21,95
turn_n	7,8783	hp_turn_n_s	6,7376	turn_n_s_xl	10,721
turn_r	6,7794	hp_turn_r_s	5,832	turn_r_s_xl	7,1342
cen_dyn	2,2975	hp_cen_dyn_s	1,7925	cen_dyn_xl	2,0088
export	10,644	hp_export_s	9,3094	export_s_xl	9,1425
import	10,283	hp_import_s	8,8655	import_s_xl	7,2283
pr_prz	28,043	hp_pr_prz_s	24,88	pr_prz_s_xl	25,432
d_prz	36,944	hp_d_prz_s	30,051	d_prz_s_xl	23,114
pr_cen	11,523	hp_pr_cen_s	10,443	pr_cen_s_xl	11,442
pr_bud	12,015	hp_pr_bud_s	9,6355	pr_bud_s_xl	11,995
d_bud	23,934	hp_d_bud_s	15,469	d_bud_s_xl	23,919
dax	23,947	hp_dax_s	17,65	dax_s_xl	23,502

Source: Author's own study



Table. 2. Time series outliers after removing seasonality (nazwa\_s) (04/1992 - 09/2014)

steel_s	coal_s	cement_s	electri_s	cars_s	prod_s	inv_s
33 IO (12 1994)	250 IO ( 1 2013)	59 IO ( 2 1997)	237 AO (12 2011)	34 IO ( 1 1995)	202 TC ( 1 2009)	202 IO ( 1 2009)
8 AO (11 1992)	33 AO (12 1994)	70 TC ( 1 1998)	130 AO ( 1 2003)	269 IO ( 8 2014)	118 AO ( 1 2002)	33 AO (12 1994)
45 AO (12 1995)	262 IO ( 1 2014)	35 IO ( 2 1995)	110 AO ( 5 2001)		214 LS ( 1 2010)	215 LS ( 2 2010)
216 IO ( 3 2010)	202 TC ( 1 2009)	58 IO ( 1 1997)	239 IO ( 2 2012)		33 AO (12 1994)	225 IO (12 2010)
112 AO ( 7 2001)	264 AO ( 3 2014)	202 TC ( 1 2009)	132 TC ( 3 2003)		64 AO ( 7 1997)	203 AO ( 2 2009)
40 AO ( 7 1995)		39 IO ( 6 1995)	202 AO ( 1 2009)		72 AO ( 3 1998)	237 AO (12 2011)
81 AO (12 1998)		45 TC (12 1995)			99 AO ( 6 2000)	226 IO ( 1 2011)
98 AO ( 5 2000)		51 AO ( 6 1996)			199 LS (10 2008)	106 TC ( 1 2001)
226 LS ( 1 2011)		217 IO ( 4 2010)			105 TC (12 2000)	72 AO ( 3 1998)
14 IO ( 5 1993)		22 IO ( 1 1994)				
		48 AO ( 3 1996)				
		36 LS ( 3 1995)				
		62 IO ( 5 1997)				
		69 LS (12 1997)				
		32 AO (11 1994)				
row_s	consum_s	employ_s	unemploy_s	vacan_s	turn_n_s	turn_r_s
110 IO ( 5 2001)	33 IO (12 1994)	189 LS (12 2007)	120 LS ( 3 2002)	156 AO ( 3 2005)	201 LS (12 2008)	215 LS ( 2 2010)
226 AO ( 1 2011)	40 TC ( 7 1995)	177 LS (12 2006)	11 TC ( 2 1993)	220 LS ( 7 2010)	231 IO ( 6 2011)	262 TC ( 1 2014)
95 AO ( 2 2000)	94 AO ( 1 2000)	10 AO ( 1 1993)	118 LS ( 1 2002)	58 AO ( 1 1997)	10 TC ( 1 1993)	72 AO ( 3 1998)
108 AO ( 3 2001)	42 IO ( 9 1995)	202 LS ( 1 2009)	153 AO (12 2004)	178 IO ( 1 2007)		61 IO ( 4 1997)
185 AO ( 8 2007)	203 IO ( 2 2009)	210 LS ( 9 2009)	217 LS ( 4 2010)	19 AO (10 1993)		
254 AO ( 5 2013)		45 LS (12 1995)	70 LS ( 1 1998)	267 LS ( 6 2014)		
232 IO ( 7 2011)		226 LS ( 1 2011)	35 LS ( 2 1995)	190 LS ( 1 2008)		
101 AO ( 8 2000)		51 AO ( 6 1996)	58 TC ( 1 1997)			

204 AO ( 3 2009)		32 AO (11 1994)	203 AO ( 2 2009)				
		70 LS ( 1 1998)					
		63 AO ( 6 1997)					
		11 AO ( 2 1993)					
cen_dyn_s	export_s	import_s	pr_prz_s	d_prz_s	pr_cen_s	pr_bud_s	
262 IO ( 1 2014)	43 TC (10 1995)	243 TC ( 6 2012)	202 IO ( 1 2009)	205 IO ( 4 2009)	32 AO (11 1994)	235 AO (10 2011)	
265 TC ( 4 2014)	247 AO (10 2012)	202 IO ( 1 2009)	199 IO (10 2008)	228 LS ( 3 2011)		177 TC (12 2006)	
202 LS ( 1 2009)	45 AO (12 1995)	238 TC ( 1 2012)		55 AO (10 1996)		77 TC ( 8 1998)	
133 TC ( 4 2003)	98 AO ( 5 2000)	241 AO ( 4 2012)		31 AO (10 1994)		225 TC (12 2010)	
200 IO (11 2008)	44 AO (11 1995)	244 TC ( 7 2012)		200 IO (11 2008)			
266 IO ( 5 2014)	31 LS (10 1994)	216 IO ( 3 2010)		162 IO ( 9 2005)			
208 AO ( 7 2009)	134 AO ( 5 2003)	98 IO ( 5 2000)		201 IO (12 2008)			
	175 AO (10 2006)	240 IO ( 3 2012)		17 IO ( 8 1993)			
	97 TC ( 4 2000)	214 AO ( 1 2010)		213 LS (12 2009)			
	253 AO ( 4 2013)	134 AO ( 5 2003)					
	137 AO ( 8 2003)	108 IO ( 3 2001)					
d_bud_s	dax_s						
105 AO (12 2000)	199 IO (10 2008)						
54 AO ( 9 1996)	132 TC ( 3 2003)						
	114 TC ( 9 2001)						
	245 LS ( 8 2012)						
	79 AO (10 1998)						
	246 AO ( 9 2012)						
	241 LS ( 4 2012)						
	197 TC ( 8 2008)						

Source: Author's own study

**Table 3. Ex post mean squared forecast error in the verification interval (02 - 09/2014) of the time series of economic dynamics of Germany, after removing from them a seasonal component.**

nazwa_s	p, q	RMSE (appropriate)	RMSE (conditional)
steel_s	2, 2	1,2262	1,3123
coal_s	1, 0	15,578	15,567
cement_s	1, 1	4,1014	4,0934
electri_s	3, 3	7,4173	7,3284
cars_s	1, 1	11,983	12,074
prod_s	2, 2	4,4589	4,4557
inv_s	2, 2	5,7825	4,8233
row_s	2, 2	3,3283	2,9179
consum_s	1, 1	2,7032	2,6430
employ_s	3, 2	1,1236	1,1543
unemploy_s	3, 3	1,0041	1,0000
vacan_s	1, 0	10,853	10,792
turn_n_s	2, 3	2,5178	2,5435
turn_r_s	3, 3	4,9520	5,1338
cen_dyn_s	3, 2	7,8634	8,2474
cen_dyn_s_stat	3, 2	2,4739	2,5207
export_s	3, 3	4,8491	5,1191
import_s	3, 3	3,3552	4,3137
pr_prz_s	2, 2	5,4730	8,1967
d_prz_s	2, 2	5,8822	6,0190
pr_cen_s	3, 1	3,0159	4,3318
pr_cen_s_stat	3, 1	1,3009	1,6024
pr_bud_s	3, 2	5,1142	5,4118
d_bud_s	3, 3	3,7736	3,9567
dax_s	2, 0	2,5874	2,6301

Source: Author's own study

**Table 4. Ex post mean squared forecast error in verification interval (02 - 09/2014) of the time series of economic dynamics of Germany after using Hodrick – Prescott filter.**

nazwa_s	p, q	RMSE (appropriate)	RMSE (conditional)
hp_steel_s	2, 2	3,2923	1,6622
hp_coal_s	2, 3	12,151	13,107
hp_cement_s	1, 0	4,4766	6,2968
hp_electri_s	1, 1	4,5644	4,5584
hp_cars_s	1, 2	11,283	11,269
hp_prod_s	2, 2	3,0343	2,8196
hp_inv_s	3, 3	4,1427	4,4994
hp_row_s	2, 2	2,5543	2,0521
hp_consum_s	1, 1	1,8406	1,8730
hp_employ_s	1, 0	0,2937	0,3637
hp_unemploy_s	1, 1	1,6468	1,8400
hp_vacan_s	1, 0	11,279	11,659
hp_vacan_s_stat	1, 0	4,0848	4,1560
hp_turn_n_s	3, 3	4,0832	4,3251
hp_turn_r_s	3, 3	6,1971	6,4223
hp_cen_dyn_s	2, 1	4,4671	6,2968
hp_cen_dyn_s_stat	2, 1	2,0756	2,2253
hp_export_s	2, 2	2,8215	2,9591
hp_import_s	2, 2	3,1495	3,5864
hp_pr_prz_s	2, 2	8,5057	6,6401
hp_d_prz_s	2, 2	3,3024	3,2857
hp_pr_cen_s	3, 1	2,0915	2,3037
hp_pr_bud_s	1, 0	5,2911	5,2786
hp_d_bud_s	3,3	2,9493	2,8094
hp_dax_s	2,1	4,0448	2,9384

Source: Author's own study

**Table 5. Ex post mean squared forecast error in verification interval (02 - 09/2014) of the time series of economic dynamics of Germany after using the TRAMO/SEATS method.**

nazwa_s_xl	(p, d, q) (ps, ds, qs)	RMSE (appropriate)	RMSE (conditional)
steel_s_xl	(0, 1, 1) (0, 0, 1)	2,6386	2,86
coal_s_xl	(0, 1, 2) (0, 0, 1)	11,938	11,777
cement_s_xl	(2, 2, 1) (0, 0, 1)	4,3302	3,7176
electri_s_xl	(1, 0, 1) (0, 0, 1)	7,3385	7,3278
cars_s_xl	(2, 1, 0) (0, 0, 1)	4,4626	4,4628
prod_s_xl	(1, 1, 0) (0, 0, 1)	3,8221	3,7634
inv_s_xl	(0, 1, 1) (0, 0, 1)	4,8696	4,8566
row_s_xl	(0, 1, 1) (0, 0, 1)	1,689	1,6845
consum_s_xl	(2, 2, 1) (0, 0, 1)	1,6857	1,7002
employ_s_xl	(0, 1, 0) (0, 0, 1)	0,1377	0,17834
unemploy_s_xl	(1, 1, 0) (0, 0, 1)	1,8383	1,5736
vacan_s_xl	(0, 1, 3) (0, 0, 1)	3,1774	3,0923
turn_n_s_xl	(2, 1, 0) (1, 0, 1)	3,9358	4,0596
turn_r_s_xl	(2, 1, 0) (0, 0, 1)	3,5976	3,5921
cen_dyn_s_xl	(1, 1, 1) (0, 0, 1)	3,132	3,1295
cen_dyn_s_xl_stat	(1, 1, 1) (0, 0, 1)	0,68968	0,69174
export_s_xl	(3, 1, 1) (1, 0, 1)	3,8862	3,2973
import_s_xl	(3, 1, 1) (0, 0, 1)	2,4307	2,4108
pr_prz_s_xl	(2, 1, 0) (0, 0, 1)	6,0316	5,7823
d_prz_s_xl	(2, 2, 0) (0, 0, 1)	2,7366	2,8219
pr_cen_s_xl	(3, 1, 1) (0, 0, 1)	4,7979	4,78
pr_bud_s_xl	(0, 1, 0) (0, 0, 1)	5,7237	5,8048
d_bud_s_xl	(0, 1, 1) (0, 0, 1)	9,0827	9,154
dax_s_xl	(0, 1, 1) (0, 0, 1)	4,2965	4,3018

Source: Author's own study

## Bibliography

- Athreya K. B., Lahiri S. N. (2006), *Measure Theory and Probability Theory*, Springer Science + Business Media LLC, New York.
- Berndt E. R., Hall B. H., Hall R. E., Hausman J.A., (1974), *Estimation and Inference in Non-Linear Structural Models*, „Annals of Economic and Social Measurement”, vol. 3.
- Brandt S., (1999), *Analiza danych. Metody statystyczne i obliczeniowe (tłumaczenie L. Szmanowski)*, wyd. drugie zmienione, Wyd. Naukowe PWN, Warszawa.
- Chatfield Ch., (2000), *Time-series forecasting*, Chapman & Hall/CRC, London.
- Clements M. P., Hendry D. F., (2004), *Forecasting economic time series*, Cambridge University Press, Cambridge.
- Clements M. P., Hendry D. F., (2001), *Forecasting Non-stationary Economic Time Series*, The MIT Press, Cambridge.
- Chow G. C., (1995), *Ekonometria (przeład W. Jurek)*, Wyd. Naukowe PWN, Warszawa.
- Chukwu E., N., (2003), *Optimal Control of the Growth of Wealth of Nations*, Taylor & Francis, London and New York.
- DeLurgio S., A., (1998) *Forecasting Principles and Applications*, International Edition, Irwin McGraw - Hill, New York.
- Edgeworth F., Y., (1888), *On a New Method of Reducing Observations Relating to Several Quantities*, „Philosophical Magazine”, vol. 25.
- Ekonometria, (Pod red. M. Gruszczyńskiego i M. Podgórskiej), (2003), SGH, Warszawa.
- Green W., H., (2000), *Econometric Analysis*, Fourth Edition, Prentice Hall International, Inc, New Jersey.
- Jajuga K., (1993), *Statystyczna analiza wielowymiarowa*, PWN, Warszawa.
- Kosiorowski D., (2012a), *Statystyczne funkcje głębi w odpornej analizie ekonomicznej*, Wydawnictwo UE w Krakowie, Kraków.
- Kosiorowski D., (2012b), *Wstęp do statystyki odpornej. Kurs z wykorzystaniem środowiska R*, Wydawnictwo UE w Krakowie, Kraków.
- Kufel T., (2007), *Ekonometria. Rozwiązywanie problemów z wykorzystaniem programu GRETL*, Wyd. Naukowe PWN, Warszawa.
- Lem S., (2010), *Filozofia przypadku. Literatura w świetle empirii*, Dzieła tom XXVI, Biblioteka Gazety Wyborczej, Warszawa.
- Lopez - Pintado S., Romo J., (2006), *Depth-Based Classification for Functional Data [in:] Series in Discrete Mathematics and Theoretical Computer Science*, Liu R. Y., Serfling R., Souvaine D. L. (eds.), vol. 72, AMS.
- Lopez - Pintado S., Romo J., (2009), *On the Concept of Depth for Functional Data*, „Journal of the American Statistical Association”, vol. 104(486).
- Maddala G., S., (2006), *Ekonometria (Red. naukowy przekładu M. Gruszczyńskiego)*, Wyd. Naukowe PWN, Warszawa.
- Maronna R. A., Martin R., D., Yohai V., J., (2006), *Robust Statistics - Theory and Methods*, John Wiley & Sons Chichester.
- Prognozowanie gospodarcze. Metody i zastosowanie, (2001), (Pod red.

M. Cieślak), Wyd. Naukowe PWN, Warszawa.

Statystyczna analiza danych z wykorzystaniem programu R, (2009), (Red. naukowa M. Walesiak, E. Gatnar), Wyd. Naukowe PWN, Warszawa.

Studemundt A. H., (2001), Using Econometrics. A Practical Guide, Fourth Edition, Addison Wesley Longman, Inc., New York.

## **Prognozowanie dynamiki gospodarczej Niemiec z pomocą modeli warunkowych (1992-2014)**

### **Abstrakt**

Dynamikę gospodarczą Niemiec w długich okresach czasu cechuje duże zróżnicowanie. Dotyczy to wielu szeregów czasowych: w pewnych okresach wykazują one dużą zmienność, która następnie przechodzi w fazę stabilizacji i zastoju. Stwarza to określone trudności w długookresowym prognozowaniu dynamiki gospodarczej. Celem podjętych badań była próba ustalenia efektywności progностycznej modelowania warunkowego, próba odpowiedzi na pytanie czy warunkowe błędy prognoz długoterminowych są znacząco mniejsze od błędów bezwarunkowych.

Badanie pokazało, że warunkowe błędy prognozy ex post nie różnią się znacząco od błędów bezwarunkowych. Zmniejszający się błąd prognozy ex post dla poszczególnych procesów gospodarczych Niemiec (przy założeniu występowania tylko wyrazu wolnego w procedurze ARMA) jest silniej skorelowany z procedurą filtrowania szeregów ekonomicznych aniżeli z zastosowaniem uwarunkowanej metody największej wiarygodności i procedur odpornych na wartości odstające. Zależność między malejącym średnim kwadratowym błędem prognozy ex post i wykorzystaniem warunkowej MNW występuje natomiast prognozach ARMAX (z procesami egzogenicznymi), przy czym minimalny warunkowy błąd prognozy wystąpił dla danych przefiltrowanych z pomocą filtra Hodricka - Prescottta. Na uwagę zasługuje względnie wysoka efektywność progностyczna odpornej estymacji regresji kwantylowej dla danych zli-nearyzowanych z pomocą metody TRAMO/SEATS.

**Słowa kluczowe:** procedury odporne, regresja kwantylowa, ARMA, ARMAX, filtr Hodricka - Prescottta, TRAMO/SEATS

